Barotropic Interaction between Planetary– and Synoptic–Scale Waves during the Life Cycles of Blockings①

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ABSTRACT

In this paper, in an equivalent barotropic framework a new forced nonlinear Schroedinger equation is proposed to examine the interaction between the planetary–scale waves and the localized synoptic–scale eddies upstream. With the help of the perturbed inverse scattering transform method, nonlinear parameter equations can be derived to describe the evolution of the dipole soliton amplitude, frequency, group velocity and phase under the forcing of localized synoptic–scale eddies. The numerical solutions of these equations predict that in the interaction between the weak dipole soliton (weak incipient dipole anomaly) and the synoptic–scale eddies, only when the high–frequency eddies themselves have a moderate parameter match they can near resonantly enhance a quasi–stationary large–amplitude split flow. The instantaneous total streamfunction field (the sum of background westerly wind, envelope Rossby soliton and synoptic–scale waves) is found to be very similar to the observed Berggren–type blocking on the weather map(Berggren et al. 1949). The role of synoptic–scale eddies is to increase the amplitude of large–scale dipole anomaly flow, and to decrease its group velocity, phase velocity and zonal wavenumber so that the dipole anomaly system can be amplified and transferred from dispersive system to very weak dispersive one. This may explain why and how the synoptic–scale eddies can reinforce and maintain vortex pair block. Furthermore, it is clearly found that during the prevalence of the vortex pair block the synoptic–scale eddies are split into two branches around the vortex pair block due to the feedback of amplified dipole block.

Key words: Envelope Rossby soliton, Blocking, Synoptic–to–planetary scale interaction

1. Introduction

In the mid–high latitudes, "blocking flow" is an important part of the low–frequency variability of the atmospheric circulation. Owing to its longevity and large amplitude, a blocking flow configuration can usually cause prolonged anomalous weather situations over certain extratropical regions. Therefore, it has been a primary interest of many synoptic and dynamical meteorologists. In the past decades, the interaction of planetary waves with migratory, synoptic–scale waves has been clearly recognized to be an important mechanism for the

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formation and maintenance of blocking (Berggren et al., 1949; Green, 1977; Illari and Marshall, 1983; Dole, 1986; Holopainen and Fortelius, 1987; Colucci, 1985, 1987; Crum and Stevens, 1988; Nakamura et al., 1990, 1993, 1997). In the interaction process, the high frequency synoptic–scale waves exert a positive feedback that acts to reinforce the blocking mainly through vorticity fluxes (Green, 1977; Hansen and Chen, 1982; Shutts, 1986; Tsou and Smith, 1990; Nakamura et al., 1990, 1997; Lupo and Smith, 1995). The viewpoint has drawn strong support from numerical experiments (Shutts, 1983; Haines and Marshall, 1987; Vautard et al., 1988a,b).

Upon observation, Colucci (1985, 1987) had shown that the formation of atmospheric blocking may be understood as a response of the planetary waves to synoptic–scale perturbations, which act as sources of energy and vorticity for the incipient blocks. Furthermore, she suggested that whether a blocking structure occurs and what type of blocking structure occurs may depend critically upon the amplitude of existing planetary waves and the phase of the waves relative to the synoptic–scale eddies. These observational studies raise very interesting and important questions: How do incipient planetary–scale blocking waves change during the passage of imbedded synoptic–scale disturbances and how are the synoptic–scale waves modulated by the amplified planetary blocking wave? These problems need to be further explored theoretically. In this theoretical study, we try to investigate these problems. It should be pointed out that because the blocking usually possesses barotropic structure and the synoptic–scale waves become less baroclinic as they approach the block (Nakamura and Wallace, 1993), as a simplest case we can use an equivalent barotropic model to examine our problems (Shutts 1983; Nakamura et al. 1997). In this case, the relevant dynamics of atmospheric blocking which arises due to the interaction of the synoptic–scale waves with the planetary–scale environment can be elucidated clearly.

In the present work, an envelope Rossby soliton (slowly modulated wave) is regarded as an incipient planetary–scale blocking wave when it encounters the synoptic–scale waves (eddies) upstream (or external forcing) (Luo, 1999b; 2000). Using a perturbation expansion method, a forced nonlinear Schroedinger equation can be derived to describe the interaction between an incipient planetary–scale blocking wave and the synoptic–scale waves upstream. In order to simplify our problem, we may assume that both antecedent planetary–scale waves and synoptic–scale waves exist in a weak background flow before a high–amplitude blocking anomaly is established, while the synoptic–scale eddies are assumed to consist of two Rossby waves with the time scales less than one week even though they are rather crude in comparison with the observations. However, as a highly simplified, theoretical problem we can do so. For our purpose, the difference between the zonal wavenumbers of two Rossby waves less than one week is assumed to be small enough so that the zonal wavenumber and frequency of the planetary–scale eddy vorticity forcing induced by these synoptic–scale Rossby waves are close to that of the studied incipient planetary–scale blocking wave. In this case, these synoptic–scale waves upstream can near resonantly force the incipient planetary–scale blocking wave. As demonstrated by Pierrehumbert and Malguzzi (1984) and Colucci (1985), blocking may be understood as a near–resonant response of the planetary waves to synoptic–scale disturbances. Based on these considerations, a force nonlinear Schroedinger equation can be derived to describe the planetary blocking wave modulation caused by the synoptic–scale eddies. When the planetary–scale blocking wave is amplified, the interaction between the amplified planetary–scale blocking wave and synoptic–scale waves will induce the modulation of the synoptic–scale waves themselves. This idea can basically reflect the planetary–to–synoptic scale interaction during the life cycles of observed blocking events (Colucci, 1985; Nakamura et al., 1990, 1993, 1997).

The present paper is organized as follows: In Section 2 we present the derivation of a forced nonlinear Schroedinger equation. The horizontal structure of the free dipole envelope soliton is presented in Section 3. Section 4 presents the main features of the planetary–scale dipole envelope Rossby soliton and the synoptic–scale eddies during their interaction. The conclusion and discussions are given in Section 5.

2. The barotropic model and the governing equation

Many investigators had confirmed that the strong interaction between planetary– and synoptic–scale waves took place during the life cycles of blockings (Berggren et al., 1949; Rex, 1950; Green, 1977; Colucci, 1985, 1987; Crum and Stevens, 1988; Neilley, 1990; Nakamura et al., 1990, 1993, 1997). Strictly speaking, although the synoptic–scale waves are purely baroclinic, the conversion of their kinetic energy to planetary wave, however, is barotropic (Frisius et al. 1998). In order to clearly clarify the interaction, both planetary blocking wave and synoptic–scale waves are assumed to be purely barotropic. This assumption is acceptable for studying the physical mechanism of blocking events associated with synoptic–scale waves. Unfortunately, no theoretical work is focused on investigating how the planetary blocking wave changes during the passage of imbedded synoptic–scale disturbances upstream and how the synoptic–scale waves are modulated. In the present paper, we will use the equivalent barotropic model (McWilliams, 1980; Shutts, 1983) to investigate this problem.

The nondimensional, nondissipative and equivalent barotropic vorticity equations describing the coupling between the planetary–scale waves and synoptic–scale eddies having a constant background westerly wind can be easily written as (Luo, 1999b)

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi-F\psi)+J(\psi,\nabla^2\psi)+(\beta+F\bar{u})\frac{\partial\psi}{\partial x}=-J(\psi',\nabla^2\psi')_P, \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi'-F\psi')+J(\psi',\nabla^2\psi')_S+(\beta+F\bar{u})\frac{\partial\psi'}{\partial x}=-J(\psi',\nabla^2\psi)-J(\psi,\nabla^2\psi'), \quad (2)$$

where the atmospheric streamfunction $\psi_T=-\bar{u}y+\psi+\psi'$ has been decomposed into three parts: basic flow $\bar{u}$, planetary–scale part ψ and synoptic–scale part ψ', and the other notation can be found in Shutts (1983). In the above–mentioned two equations, the eddy vorticity forcing $-J(\psi',\nabla^2\psi')$ has been decomposed into two parts: planetary–scale component $-J(\psi',\nabla^2\psi')_P$ and synoptic–scale component $-J(\psi',\nabla^2\psi')_S$. Strictly speaking, this decomposition is rather crude, but basically admissible for interpreting the physical mechanism of the planetary–to–synoptic scale interaction during the life cycles of blockings. It is also easy to find that $-J(\psi',\nabla^2\psi')_P$ should be placed in Eq.(1), while $-J(\psi',\nabla^2\psi')_S$ should be placed in Eq. (2). Note that $-J(\psi',\nabla^2\psi')_P$ could be naturally regarded as a source of energy and vorticity for planetary blocking waves (Shutts, 1983; Colucci, 1985; Vautard et al., 1988a,b; Nakamura et al., 1990, 1993, 1997).

The planetary–scale flow is contained within a beta–channel on whose boundaries

$$\frac{\partial\psi}{\partial x}=0, \quad y=0, \quad Ly, \quad (3)$$

while the zonally averaged part of the streamfunction, $\bar{\psi}(y,t)$, must satisfy (Pedlosky, 1979; Luo, 1999a)

$$\frac{\partial^2 \overline{\psi}}{\partial t \partial y} = 0, \quad y = 0, \quad Ly, \tag{4}$$

where Ly is the width of the beta–channel.

Generally speaking, in previous studies the term $-J(\psi',\nabla^2\psi')_P$ which arises from the synoptic–scale waves is usually specified as an external forcing (Pierrehumbert and Malguzzi, 1984; Nakamura et al., 1997), but this treatment is unable to investigate how the migratory, synoptic–scale eddies are modulated by the amplified blocking wave. As pointed out by Nakamura et al. (1997), the synoptic–scale eddies themselves are modulated as the blocking strengthens due to the forcing of the synoptic–scale waves. So far, no theoretical study is presented to describe such a interaction process. In this paper, we present a new, simple theoretical model to deal with this problem. For some of blocking cases, the relative vorticity advection term,$-J(\psi,\nabla^2\psi)$ of the planetary–scale waves is almost an order of magnitude larger than that of the planetary–scale eddy forcing term, $-J(\psi',\nabla^2\psi')_P$, induced by the synoptic–scale waves upstream (Holopainen and Fortelius 1987). In this case, if $J(\psi,\nabla^2\psi)$ has an order of ε^2, then $-J(\psi',\nabla^2\psi')_P$ should have an order of ε^2. Therefore, if the planetary–scale blocking wave can be expanded as $\psi = \varepsilon\psi_0 + \varepsilon^2\psi_1 + \cdots$, then the synoptic–scale waves should have the expansion $\psi' = \varepsilon^{3/2}\psi_1' + \varepsilon^{5/2}\psi_2' + \cdots$. In addition, to make the slowly varying envelope amplitude equation of the planetary–scale blocking wave include the planetary–scale eddy vorticity forcing term, the expansion $\psi' = \varepsilon^{3/2}\psi_1' + \varepsilon^{5/2}\psi_2' + \cdots$ for the synoptic–scale waves also should be made when the expansion $\psi = \varepsilon\psi_0 + \varepsilon^2\psi_1 + \cdots$ is made for the planetary–scale waves. In other words, if $\psi' = \varepsilon^{3/2}\psi_1' + \varepsilon^{5/2}\psi_2' + \cdots$ is allowed, the envelope amplitude equation of the blocking wave, at the order $O(\varepsilon^3)$, can include the forcing term of $-J(\psi_1',\nabla^2\psi_1')_P$. It should be pointed out that for some of blocking cases, the relative vorticity advection term of the planetary–scale waves sometimes has the same order as that of the observed planetary–scale eddy forcing term (Nakamura et al., 1997). Though our treatment here is inappropriate for these blocking cases, the basic results obtained from our model are acceptable. This can be confirmed by using the three wave quasi–resonant interaction theory, which is not presented in the paper. When both the expansions $\psi = \varepsilon\psi_0 + \varepsilon^2\psi_1 + \cdots$ and $\psi' = \varepsilon^{3/2}\psi_1' + \varepsilon^{5/2}\psi_2' + \cdots$ are substituted into Eq.(2), the following equations can be obtained as

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi_1' - F\psi_1') + (\beta + F\bar{u})\frac{\partial\psi_1'}{\partial x} = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi_2' - F\psi_2') + (\beta + F\bar{u})\frac{\partial\psi_2'}{\partial x} = -J(\psi_1',\nabla^2\psi_0) - J(\psi_0,\nabla^2\psi_1'), \tag{6}$$

where ψ_1' and ψ_2' denote the first– and second–order synoptic–scale flows of the synoptic–scale waves, respectively.

Note that Eq. (5) is a linear equation, which describes the first–order synoptic–scale flow that is not associated with the amplified planetary–scale wave ψ_0 governed by a nonlinear Schroedinger equation. However, the second–order synoptic–scale flow ψ_2' is associated with the amplified planetary–scale blocking wave ψ_0 through the interaction with the synoptic–scale waves ψ_1'. It is interesting to note that when the planetary–scale blocking wave ψ_0 is amplified by the synoptic–scale waves ψ_1', the modulation of the synoptic–

scale waves ψ' induced by the amplified planetary–scale wave is mainly described by ψ'_2. The modulation part ψ'_2 does not strongly influence the amplification of the planetary–scale blocking wave because its zonal wavelength is too short in comparison with that of the planetary–scale blocking wave. Although Shutts (1983) and Haines and Marshall (1987) have shown that migrating high–frequency fluctuations in a barotropic model are modulated and deformed by diffluent flows associated with a blocking–like or modon–like basic state, their numerical experiments, however, cannot provide a clear description of how the planetary–scale blocking wave changes during the passage of imbedded synoptic–scale disturbances. There is also a similar defect for the previous analytical studies (Malguzzi, 1993), but this defect can be avoided in our theoretical model.

In theoretical and numerical studies, we should consider the exact expression of observed synoptic–scale waves. However, it is very difficult to give in detail such an expression because of their complexity. As a theoretical study, deriving the exact expression of the synoptic–scale waves is unnecessary for the study of the physical mechanism of blocking events associated with the synoptic–scale waves. This is because the complex synoptic–scale waves will cause a great difficulty for the mathematical treatment in exploring the interaction between both the planetary–scale blocking wave and the synoptic–scale waves. It is unfavorable for the understanding of the essence of the physical mechanism of blockings. In the present paper, as a simplest case we may assume that the synoptic–scale wave ψ'_1 consists of two Rossby waves having the time scale less than one week. Without the loss of generality, the two Rossby waves can be assumed to be of the form

$$\psi'_1 = f'_0\{\exp[i(k_1 x - \omega_1 t)] - \rho\exp[i(k_2 x - \omega_2 t)]\}\sin(\frac{m}{2}y) + cc, \quad (7)$$

where $f'_0(X)$ is the amplitude, $m = -\frac{2\pi}{Ly}$, $\omega_1 = \bar{u}k_1 - \frac{(\beta + F\bar{u})k_1}{K_1^2}$, $\omega_2 = \bar{u}k_2 - \frac{(\beta + F\bar{u})k_2}{K_2^2}$, $K_i^2 = k_i^2 + m^2/4 + F$, $k_i = k_n + (-1)^i\Delta k$, $|\Delta k| << k_n$, $X = \varepsilon x$, cc denotes the complex conjugate of its preceding term, and ρ is an arbitrary constant. Note that the nondimensional wavelength for the nth Rossby wave is $Lx = 2\pi 6.371\cos(\varphi_0)/n$, which is identical to $2\pi 6.371 \times 10^6\cos(\varphi_0)/(nL)$, where $L = 10^6 m$ is the horizontal characteristic scale.

When we choose $Ly = 5$ (5000 km, the meridional wavelength in dimensional form), $F = 1.0$, $n = 10$, $\Delta k = 0.75k_0$ and $\bar{u} = 0.7$, (7 m / s in the dimensional form), the periods of the two Rossby waves at 55°N given in (7) correspond to nearly 6 and 5 days, respectively. It should be pointed out that $\Delta k = k_0$ can be also chosen, but this choice does not influence the results obtained in this paper. Moreover, following the definition of the synoptic–scale eddies proposed by Mullen (1987) and Holopainen and Fortelius (1987), the two Rossby waves having the time scale less than one weak described in (7) should be considered as the synoptic–scale eddies. If the zonal wavenumber of the planetary–scale blocking wave is chosen to be 2, its zonal wavelength is nearly 5 times greater than the zonal wavelength of the synoptic–scale waves in (7). When (7) is substituted into $-J(\psi', \nabla^2\psi')$, it is not difficult to find that $-J(\psi'_1, \nabla^2\psi'_1)_P$, only consists of planetary–scale component $k_2 - k_1$, while $-J(\psi', \nabla^2\psi')_S$, consists of synoptic–scale component $k_2 + k_1$. The amplitude of the planetary–scale component $k_2 - k_1$ is found to be nearly 10 times greater than that of synoptic–scale component $k_2 + k_1$ for the parameters above. Therefore, $-J(\psi', \nabla^2\psi')$ is dominated by planetary–scale component $-J(\psi'_1, \nabla^2\psi'_1)_P$, while the synoptic–scale component $-J(\psi', \nabla^2\psi')_S$ can be negligible. Following Yamagata (1980), Jeffrey and Kawahara (1982), Taniuti and Nishihara (1983) and Hasegawa and Kodama(1995) we can introduce the slowly varying coordinates

$$\xi = \varepsilon(x - Cgt),\ T = \varepsilon^2 t, \tag{8}$$

where $Cg = \bar{u} - \dfrac{(\beta + F\bar{u})(m^2 + F - k^2)}{(k_2 + m^2 + F)^2}$, $\gamma_0 \leqslant \varepsilon << 1.0$, and $\gamma_0 = \dfrac{U}{f_0 L} \approx 0.1$ is the local Rossby number. The transform (8) we make here implies that the amplitude of the planetary–scale blocking wave is slowly varying through the nonlinear interaction or the forcing. The zonal scale of the slowly varying envelope amplitude is several $(1/\varepsilon)$ times greater than the scale of the carrier wave k. In the actual application the singular perturbation theory is also correct even if the perturbation parameter is not rather small. This is because the choice of ε does not strongly affect the obtained results.

In the real atmosphere, the blocking flows usually exhibit zonal wavenumbers 1, 2 and 3 (Colucci et al., 1981). If $Ly = 5.0$ is chosen (5000 km in the dimensional form, presented by McWilliams, 1980 in his Fig.1), we can obtain $Ly / Lx = 0.436 \sim 0.65$ at 55°N for zonal wavenumbers 2 and 3. It is natural to conclude that the long–wave approximation $Ly / Lx << 1.0$ (or $k \to 0$) might be inappropriate for blocking systems having large meridional scale for zonal wavenumbers 2 and 3. But for zonal wavenumber 1 the long wave approximation can be crudely acceptable. However, in deriving a nonlinear Schroedinger equation, the long–wave approximation is not required. It seems that the KdV soliton is only reasonable for blocking events having a very long zonal scale (Butchart et al. 1989). On the other hand, it should be pointed out that the isolated coherent structures obtained from the KdV Rossby soliton model are nondispersive at all stages. In fact, the blocking during the mature stage can be considered as a nondispersive wave, but during the onset and decay stages it is dispersive. Therefore, the long–wave approximation distorts seriously the nature of blocking during the onset and decay stages. This is why we try to study the dynamics of forced envelope Rossby soliton.

If the solution to Eq.(1) can be expanded as

$$\psi = \varepsilon\psi_0(\xi, T, x, y, t) + \varepsilon^2\psi_1(\xi, T, x, y, t) + \varepsilon^3\psi_3(\xi, T, x, y, t) + \cdots, \tag{9}$$

then substitution of (9) into Eq.(1) yields

$$O(\varepsilon):\quad L(\psi_0) = \left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)[\nabla^2(\psi_0) - F(\psi_0)] + (\beta + F\bar{u})\frac{\partial(\psi_0)}{\partial x} = 0, \tag{10}$$

$$O(\varepsilon^2):\quad L(\psi_1) = -J(\psi_0, \nabla^2\psi_0), \tag{11}$$

$$\begin{aligned} O(\varepsilon^3):\quad L(\psi_2) = & -\Big\{\frac{\partial}{\partial T}(\nabla^2\psi_0 - F\psi_0) + 2(\bar{u} - Cg)\frac{\partial^3\psi_0}{\partial x\partial\xi^2} + \left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\frac{\partial^2\psi_0}{\partial\xi^2} + \\ & (\beta + F\bar{u})\frac{\partial\psi_1}{\partial\xi} + (\bar{u} - Cg)\frac{\partial}{\partial\xi}(\nabla^2\psi_1 - F\psi_1) + J(\psi_0, \nabla^2\psi_1) + J(\psi_1, \nabla^2\psi_0) \\ & + J\left(\psi_0, 2\frac{\partial^2\psi_1}{\partial x\partial\xi}\right) \\ & + \frac{\partial\psi_1}{\partial\xi}\frac{\partial}{\partial y}\nabla^2\psi_0 - \frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial\xi}\nabla^2\psi_1\Big\} - if_0'^2\rho\frac{(k_1 + k_2)^2(k_2 - k_1)}{4}m\exp\{i[2\Delta kx - \\ & (\omega_2 - \omega_1)t]\}\sin(my). \end{aligned} \tag{12}$$

For planetary–scale Rossby waves having dipole meridional structure, the solution to (10) can be assumed to be of the form

$$\psi_0 = A(\xi,T)\varphi_1(y)\exp[i(kx-\omega t)] + cc, \tag{13}$$

where $\varphi_1(y) = \sqrt{\frac{2}{Ly}}\sin(my)$, $\omega = \bar{u}k - \frac{(\beta + F\bar{u})k}{k^2+m^2+F}$, $k = \frac{2}{6.371\cos(\varphi_0)}$, is the zonal wavenumber of Rossby wave 2, and $A(\xi,T)$ is the complex envelope amplitude. It is clear to find that as a first–order approximation of planetary–scale waves, ψ_0 satisfies a linear Rossby wave equation. The observational study presented by Holopainen and Fortelius (1987) has shown that to the leading order approximation the time–mean flow in their blocking case behaves as a "free" stationary wave (Shutts, 1983). This indicates that the expansion solution in (9) is correct if ψ_0 has a blocking structure.

Using (12), the solution to Eq. (11) satisfying the boundary condition (4) can be obtained as

$$\psi_1 = -|A|^2\sum_{n=1}^{\infty} q_n g_n \cos(n+\frac{1}{2})my + \varepsilon\psi_2 \quad , \tag{14}$$

where $q_n = \frac{4k^2 m}{Ly\{\beta + F\bar{u} - (\bar{u} - Cg)[F + (n+\frac{1}{2})^2 m^2]\}}$ and $g_n = \frac{8}{m[4-(n+\frac{1}{2})^2]Ly}$.

It is easy to note that in Eq. (12), the interaction between the synoptic–scale eddies themselves induces planetary–scale eddy forcing component $k_2 - k_1 = 2\Delta k$. If $k_2 - k_1$ is near that of planetary–scale wave, then the eddy forcing will near–resonantly force the planetary–scale wave. A special case is that the zonal wavenumber $k_2 - k_1$ of the planetary–scale eddy forcing is assumed to be the same as that of the studied planetary–scale wave. Without the loss of generality, we can choose parameters $Ly = 5$, $F = 1.0$, $n = 10$, $\Delta k = 0.75k_0$ and $\bar{u} = 0.7$. For these parameters, we know that $\omega_2 - \omega_1 - \omega$ is small and $k - 2\Delta k = O(\varepsilon)$. Assuming $k - 2\Delta k = \varepsilon\Delta K$ and $\omega_2 - \omega_1 - \omega = \varepsilon^2\Delta\Omega$, and when (13) and (14) are substituted into Eq. (12), the non–secularity condition yields

$$i\frac{\partial A}{\partial T} + \lambda\frac{\partial^2 A}{\partial\xi^2} + \delta|A|^2 A + Gf_0'^2\exp[-i(\Delta KX + \Delta\Omega T)] = 0, \tag{15}$$

where $\lambda = \frac{[3(m^2+F)-k^2](\beta+F\bar{u})k}{(k^2+m^2+F)^3}$, $\delta = \frac{km\sum_{n=1}^{\infty}[k^2+m^2-m^2(n+\frac{1}{2})^2]q_n g_n^2}{k^2+m^2+F}$,

$G = -\sqrt{\frac{Ly}{2}}\frac{(k^2+k_1)^2(k_2-k_1)mp}{4(k^2+m^2+F)}$ and $X = \xi + \varepsilon Cgt$.

Once the solution of Eq.(15) can be obtained, the second–order synoptic–scale flow ψ'_2 can be determined from Eq.(6) and expressed as

$$\begin{aligned}\psi'_2 = &-\frac{m}{4}Q_1\sqrt{\frac{2}{Ly}}Af'_0\exp\{i[(k_1+k)x-(\omega_1+\omega)t]\}[p_1\sin(\frac{3m}{2}y)+r_1\sin(\frac{m}{2}y)]\\ &+\frac{m}{4}Q_2\sqrt{\frac{2}{Ly}}Af'_0\exp\{i[(k_2+k)x-(\omega_2+\omega)t]\}[p_2\sin(\frac{3m}{2}y)+r_2\sin(\frac{m}{2}y)]\\ &+\frac{m}{4}Q_1\sqrt{\frac{2}{Ly}}A^*f'_0\exp\{i[(k_1-k)x-(\omega_1-\omega)t]\}[s_1\sin(\frac{3m}{2}y)+h_1\sin(\frac{m}{2}y)]\end{aligned}$$

$$-\frac{m}{4}Q_2\sqrt{\frac{2}{Ly}}A^*\rho f'_0\exp\{i[(k_2-k)x-(\omega_2-\omega)t]\}[s_2\sin(\frac{3m}{2}y)+h_2\sin(\frac{m}{2}y)], \quad (16)$$

where the coefficients Q_i, p_i, r_i, s_i and $h_i(i=1,2)$ in expression (16) are real and defined in the Appendix.

It is easy to find from (15) and (16) that because the amplitude equation for A includes the term f'_0, the planetary–scale blocking wave will be deformed during the passage of the synoptic–scale waves upstream. In this process, the second–order flow ψ'_2 of the synoptic–scale waves can be induced through the interaction with the deformed planetary–scale wave. This second–order flow ψ'_2 reflects how the synoptic–scale waves are modulated by the planetary–scale wave. Therefore, if the planetary wave for A can be amplified into a typical blocking, our model presented here can basically describe the interaction between planetary– and synoptic– scale waves during the life cycles of blocking events (Colucci 1985, 1987). It should be pointed out that the previous theoretical models cannot describe this interaction (Malguzzi, 1993).

With the help of the transforms $A=B/\sqrt{\delta}$ and $\zeta=\xi/\sqrt{2\lambda}$, Eq.(17) can be rewritten in the form

$$i\frac{\partial B}{\partial T}+\frac{1}{2}\frac{\partial^2 B}{\partial \zeta^2}+|B|^2B+G\sqrt{\delta}f'^2_0\exp[-i(\Delta KX+\Delta\Omega T)]=0. \quad (18)$$

Eq.(18) is a forced nonlinear Schroedinger (NLS) equation, which describes the deformation of the planetary–scale wave caused by the synoptic–scale eddies. In a wide parameter range both λ and δ are positive for planetary–scale Rossby waves. Therefore, for the case without forcing this NLS equation, there exists an envelope soliton solution. Generally speaking, it is very difficult to obtain the exact solution of forced NLS equation. However, if the forcing term is weak, the perturbed inverse scattering transform (PIST) method can be applied to solving Eq.(18) (Hasegawa and Kodama, 1995).

If the solutions to Eq.(18) can be obtained with the help of both PIST method and fourth–order Rung–Kutta method when f'_0 is specified as a known function, we can clearly understand how the dipole envelope Rossby soliton (incipient block) is amplified by the synoptic–scale waves and how the synoptic–scale waves are modulated by the amplified envelope Rossby soliton. As the first step of our task we will give the horizontal structures of free planetary–scale dipole envelope Rossby soliton and discuss the possibility of its application to blocking events.

3. Analysis of the solutions and the horizontal structures of free planetary–scale dipole envelope Rossby solitons

Under the condition without forcing, the envelope soliton solution to Eq. (18) can be obtained as (Taniuti and Nishihara, 1983)

$$A=A_0\operatorname{sech}[\sqrt{\frac{\delta}{2\lambda}}A_0\xi]\exp(\frac{i}{2}\delta A_0^2T), \quad (19)$$

where A_0 is the value of A at $(\xi,T)=(0,0)$.

If one defines $M_0=\varepsilon A_0$, the streamfunction field of the dipole envelope Rossby soliton can be expressed as

$$\psi_e = -\bar{u}y + \tilde{\psi} = -\bar{u}y + \varepsilon\psi_0 + O(\varepsilon^2)$$

$$= -\bar{u}y + \varepsilon A_0\sqrt{\frac{2}{Ly}}\,\mathrm{sech}[\sqrt{\frac{\delta}{2\lambda}}A_0\xi]\sin(my)\exp[i(kx-\omega t+\frac{1}{2}\delta A_0^2T)] + O(\varepsilon^2) + cc$$

$$= -\bar{u}y + M_0\sqrt{\frac{2}{Ly}}\,\mathrm{sech}[\sqrt{\frac{\delta}{2\lambda}}M_0(x-Cgt)]\sin(my)\exp[ik(x-C_At)] + O(\varepsilon^2) + cc, \quad (20)$$

where $\xi = \varepsilon(x - Cgt)$ and $T = \varepsilon^2 t$ have been used, and $C_A = \omega/k - \delta M_0^2/(2k)$.

It is not difficult to find from (19) and (20) that the envelope amplitude of the dipole Rossby soliton for $\tilde{\psi}(\psi_e = -\bar{u}y + \tilde{\psi})$ possesses an isolated coherent structure of sech–shape, but $\cos k(x - C_A t)$ does not vanish. However, the streamfunction field of KdV Rossby soliton only has a nondispersive isolated structure of sech^2–shape because the long wave approximation $Ly/Lx << 1.0$ (or $k \to 0$) is used (Malguzzi, 1993 and Malanotte–Rizzoli et al., 1987; Butchart et al., 1989). The structure of the KdV Rossby soliton is solely determined by the zonal scale of the sech^2–shape. If the KdV Rossby soliton possesses a realistically blocking structure, then the zonal scale of the sech^2–shape amplitude should at least be the scale of dipole blocking. This seems to be in contradiction with the assumption $Ly/Lx << 1.0$ when Ly is sufficiently large (for example, $Ly = 5.0$ in McWilliams, 1980). As pointed out by Butchart et al. (1989), the isolated structure of the KdV Rossby soliton is mainly confined in the narrower channel. However, there does not exist such a limitation for an envelope Rossby soliton. This is because the envelope soliton ($\psi = -\bar{u}y + \tilde{\psi}$) also possesses blocking structure even if the zonal scale of the envelope amplitude A is very large even infinite (for this case A is a constant, and the solution (9) reduces to a linear Rossby wave solution) so long as A has a large amplitude state (Shutts, 1983). For example, in the resonant theory of blocking proposed by Tung and Lindzen (1979), stationary blocking can be produced through increasing wave amplitude under the resonant forcing of large–scale topography. Therefore, we think that the KdV soliton theory of blocking proposed by Malguzzi (1993) and Malanotte–Rizzoli et al. (1987) is only to consider the case for $k \to 0$ in $\psi = \varepsilon A(\xi, T)\varphi_1(y)\exp[i(kx - \omega t)] + cc$ as the shear basic flow exists, while the linear theory of blocking proposed by Tung and Lindzen (1979) is only to consider the case that $A(\xi, T)$ becomes a constant. Thus, their theories are only special cases of the envelope Rossby soliton theories studied here. Consequently, we can conclude that the envelope Rossby soliton proposed here seems to be more reasonable for blocking events than the previous theories. On the other hand, it should be pointed out that the derivation of nonlinear Schroedinger equation has an implication that the zonal scale of the envelope amplitude is several times ($1/\varepsilon$) greater than the wavelength of the carrier wave k. This is a necessary condition for the use of the perturbation expansion method. However, in the real application the perturbation expansion method can be approximately used as long as the zonal scale of the envelope amplitude is larger than the space scale of the carrier wave.

When $\delta \to 0$ (it corresponds to linear wave), (20) reduces to the linear Rossby wave in the form

$$\psi_l = -\bar{u}y + 2M_0\sqrt{\frac{2}{Ly}}\sin(my)\cos[k(x - C_At)], \quad (21)$$

where $C_A = \omega/k$.

When the carrier wave k in (20) is excluded, the streamfunction of the envelope amplitude can be expressed as

$$\psi_A = -\bar{u}y + 2M_0\sqrt{\frac{2}{Ly}}\,\mathrm{sech}\left[\sqrt{\frac{\delta}{2\lambda}}M_0(x - Cgt)\right]\sin(my). \qquad (22)$$

For the parameters $Ly = 5$, $F = 1.0$, $\bar{u} = 0.7$ and $M_0 = 0.8$, the fields of ψ_A, ψ_l and ψ_e at 55°N are depicted in Fig. 1.

It can be found from Fig. 1a that the envelope amplitude possesses an isolated structure of the sech–shape. Through the comparison with Fig.1b, we note that the zonal scale of the envelope amplitude is larger than that of the carrier wave k. This shows that the perturbation expansion carried out in the present paper is crudely acceptable. On the other hand, for a linear wave its flow pattern has a wavy structure (Fig. 1b). But for envelope Rossby soliton the wavy structure almost disappears due to the decay of sech–shape in the zonal direction (Fig. 1c). In particular, in higher latitude regions, this character is more remarkable because of $\delta/(2\lambda)$ becoming large (figures omitted). This is why local dipole blocking can be easily observed in high latitudes. If k is large, for example, for zonal wavenumber 4 ($k = 1.09$), the ψ field of the envelope Rossby soliton for zonal wavenumber 4 has a strong wavy structure, but it doesn't belong to blocking flow pattern (figures omitted). In addition, we note that Fig. 1c at day 0 is also similar to the equivalent modons and KdV–type soliton solutions obtained by McWilliams (1980) and Malanotte–Rizzoli and Malguzzi (1987), respectively. Different point is that both the modons and KdV–type soliton are nondispersive, but the envelope soliton studied here is dispersive due to $Cg - C_A \neq 0$. Although the dispersiveness of the free envelope Rossby soliton system cannot be regarded as an advantage against the KdV soliton, it just reflects such a fact that when the external forcing such as synoptic–scale eddies and so on are excluded, no blocking flow can be maintained. As demonstrated by a number of numerical experiments, blocking system cannot be established if the external forcing is excluded. This sufficiently shows that blocking system originates from the combined role of external forcing and nonlinearity. As pointed out by the author hereafter, the dispersive envelope soliton block may become weak dispersion even non–dispersion system by adding the forcing of

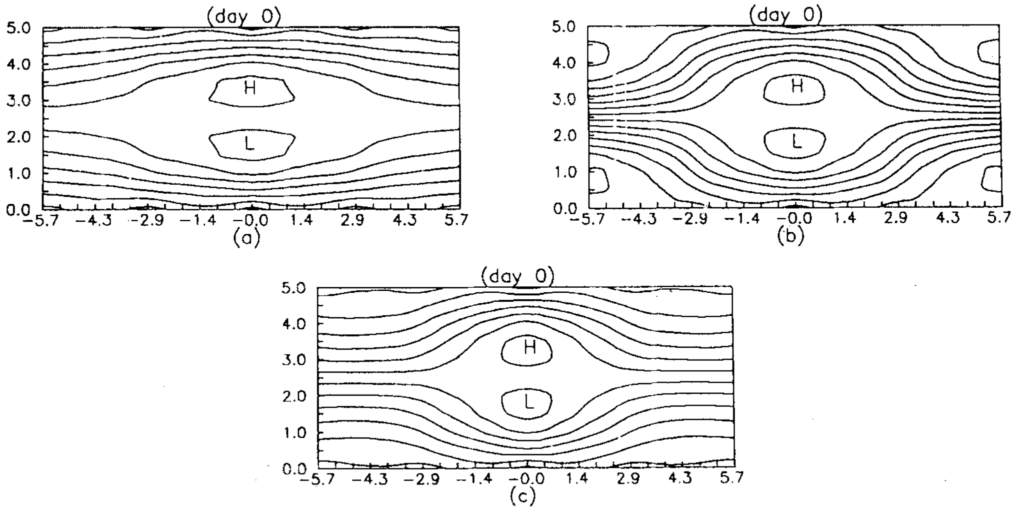

Fig. 1. Structures of ψ_A, ψ_l and ψ_e for free planetary–scale dipole envelope Rossby soliton at 55°N for $M_0 = 0.8$: (a) ψ_A field; (b) ψ_l field; (c) ψ_e field. Contour interval 0.3.

synoptic–scale eddies during the amplifying of envelope amplitude. In other words, the onset of the dipole block is a transfer of an amplified envelope Rossby soliton from dispersion to weak dispersion even non–dispersion. This process can at least explain how the synoptic–scale waves reinforce and maintain the dipole block. At the same time, one can know how the synoptic–scale waves are modulated by the dipole block. Unfortunately, previous theoretical studies cannot describe the interaction between planetary– and synoptic–scale waves during the life cycle of blocking. In the next section, based on our theoretical model we will present the behaviors of planetary– and synoptic–scale waves during the life cycle of blocking.

4. The numerical results and the behaviors of planetary– and synoptic–scale waves during the life cycle of blocking

In using the PIST method of the soliton perturbation theory, the forcing term in (18) is usually regarded as a perturbation of the formal NLS equation. The general form of the soliton solution with parameters depending on the slowly varying time is assumed to be of the form (Hasegawa and Kodama 1995)

$$B(\zeta,T)=\eta(T)\mathrm{sech}\{\eta(T)[\zeta-Z_0(T)]\}\exp\{-i\kappa(T)[\zeta-Z_0(T)]+i\theta(T)\}, \tag{23}$$

where $\eta(T)$, $\kappa(T)$, $Z_0(T)$ and $\theta(T)$ are the envelope amplitude, frequency, group velocity and phase of the soliton.

Nakamura and Wallace (1990, 1993) analyzed changes in the amplitude of synoptic–scale baroclinic waves in the composite maps for a number of cases of blocking onset, and found that the local intensity of these synoptic–scale waves should be expressed in terms of an " envelope function", a measure of the time–averaged temporal variance associated with the waves. Based on this fact, without loss of generality f'_0 may be assumed to be of the form

$$f'_0=a'_0\exp[-\gamma(X+\varepsilon b)^2], \tag{24}$$

where a'_0 is the amplitude of the localized synoptic–scale waves, $\gamma>0$ and b denotes the position of the localized eddy forcing. Here (24) is designed to imitate the high–frequency synoptic–scale waves over the Atlantic storm track, upstream of vortex pair block (Holopainen and Fortelius, 1987). The choice of $b>0$ means that the localized synoptic–scale waves are located upstream of a splitting jetstream (high–over–low dipole) described by a dipole envelope Rossby soliton. The latter discussion is based on this consideration.

If one defines the new variables $\varepsilon\eta(\varepsilon^2 t)=\sqrt{\delta}M(t)$, $Z(t)=\dfrac{Z_0(\varepsilon^2 t)\sqrt{2\lambda}}{\varepsilon}$, $\varepsilon\kappa(\varepsilon^2 t)=K(t)$, and $\theta(\varepsilon^2 t)=P(t)$, in terms of the PIST method the following soliton parameter equations can be obtained as

$$\frac{dM}{dt}=G\sqrt{\frac{\delta}{2\lambda}}Ma_0^2\int_{-\infty}^{\infty}F(x',t)\mathrm{sech}(\mathscr{P})\sin(\Theta)dx', \tag{25}$$

$$\frac{dK}{dt}=-G\frac{\delta}{\sqrt{2\lambda}}Ma_0^2\int_{-\infty}^{\infty}F(x',t)\mathrm{sech}(\mathscr{P})\tanh(\mathscr{P})\cos(\Theta)dx', \tag{26}$$

$$\frac{dZ}{dt} = -K\sqrt{2\lambda} + \sqrt{\frac{\delta}{2\lambda}} G a_0^2 M \int_{-\infty}^{\infty} F(x',t) x' \mathrm{sech}(\mathscr{R}) \sin(\Theta) dx', \tag{27}$$

$$\frac{dP}{dt} = -\frac{K^2 - \delta M^2}{2} - \frac{K}{\sqrt{2\lambda}}\frac{dZ}{dt} + \sqrt{\frac{\delta}{2\lambda}} G M a_0^2 \int_{-\infty}^{\infty} F(x',t) \mathrm{sech}(\mathscr{R}) \cos(\Theta)[1 - \mathscr{R}\tanh(\mathscr{R})] dx' \tag{28}$$

where $F(x',t) = \exp[-2\gamma\varepsilon^2(x' + Cgt + Z + b)^2]$, $\mathscr{R} = \sqrt{\frac{\delta}{2\lambda}} Mx'$ $a_0 = \varepsilon a'_0$, $\xi = \varepsilon(x - Cgt)$, $X = \varepsilon x$, $T = \varepsilon^2 t$ and $\Theta(x',t) = (k - 2\Delta k)(x' + Cgt + Z) + (\omega_2 - \omega_1 - \omega)t - \frac{K}{\sqrt{2\lambda}} x' + P$.

The streamfunction of the amplified planetary–scale dipole soliton during the interaction with the localized synoptic–scale waves can be easily expressed as

$$\psi = -\bar{u}y + \sqrt{\frac{2}{Ly}} M(t) \mathrm{sech}\{\sqrt{\frac{\delta}{2\lambda}} M(t)[x - Cgt - Z(t))]\} \times$$
$$\exp\{i\{kx - \omega t - \frac{K(t)}{\sqrt{2\lambda}}[x - Cgt - Z(t)] + P(t)\}\}\sin(my) + \varepsilon^2\psi_1 + cc. \tag{29}$$

When the feedback of the deformed planetary–scale dipole soliton on the localized synoptic–scale waves is considered, the streamfunction of the deformed synoptic–scale waves can be obtained as

$$\psi' = 2a_0 \exp[-\gamma\varepsilon^2(x+b)^2]\{\cos(k_1 x - \omega_1 t) - \rho\cos(k_2 x - \omega_2 t)]\sin(\frac{m}{2}y)$$
$$-\frac{m}{2}Q_1\sqrt{\frac{2}{Ly}} a_0 M(t)\exp[-\gamma\varepsilon^2(x+b)^2]\mathrm{sech}\{\sqrt{\frac{\delta}{2\lambda}} M(t)[x - Cgt - Z(t)]\} \times$$
$$\cos\{(k_1 + k)x - (\omega_1 + \omega)t - \frac{K(t)}{\sqrt{2\lambda}}[x - Cgt - Z(t)] + P(t)\}[p_1 \sin(\frac{3m}{2}y) + r_1 \sin(\frac{m}{2}y)]$$
$$+\frac{m}{2}Q_2\sqrt{\frac{2}{Ly}} a_0 \rho M(t)\exp[-\gamma\varepsilon^2(x+b)^2]\mathrm{sech}\{\sqrt{\frac{\delta}{2\lambda}} M(t)[x - Cgt - Z(t)]\} \times$$
$$\cos\{(k_2 + k)x - (\omega_2 + \omega)t - \frac{K(t)}{\sqrt{2\lambda}}[x - Cgt - Z(t)] + P(t)\}[p_2 \sin(\frac{3m}{2}y) + r_2 \sin(\frac{m}{2}y)]$$
$$+\frac{m}{2}Q_1\sqrt{\frac{2}{Ly}} a_0 \rho M(t)\exp[-\gamma\varepsilon^2(x+b)^2]\mathrm{sech}\{\sqrt{\frac{\delta}{2\lambda}} M(t)[x - Cgt - Z(t)]\} \times$$
$$\cos\{(k_1 - k)x - (\omega_1 - \omega)t + \frac{K(t)}{\sqrt{2\lambda}}[x - Cgt - Z(t)] - P(t)\}[s_1 \sin(\frac{3m}{2}y) + h_1 \sin(\frac{m}{2}y)]$$
$$-\frac{m}{2}Q_2\sqrt{\frac{2}{Ly}} a_0 \rho M(t)\exp[-\gamma\varepsilon^2(x+b)^2]\mathrm{sech}\{\sqrt{\frac{\delta}{2\lambda}} M(t)[x - Cgt - Z(t)]\} \times$$
$$\cos\{(k_2 - k)x - (\omega_2 - \omega)t + \frac{K(t)}{\sqrt{2\lambda}}[x - Cgt - Z(t)] - P(t)\}[s_2 \sin(\frac{3m}{2}y) + h_2 \sin(\frac{m}{2}y)]. \tag{30}$$

Because the planetary–scale dipole envelope Rossby soliton possesses a vortex pair block structure, Eqs.(25)–(28) actually describe the deformation of a planetary–scale vortex pair block (dipole envelope Rossby soliton) caused by localized synoptic–scale waves. If the solutions of these equations can be derived, one can explore how the synoptic–scale waves reinforce and maintain vortex pair block. In general, the analytical solutions of Eqs.(25)–(28) are impossible. But, if the initial data are prescribed, their numerical solutions of these parameter equations can be obtained by the fourth–order Rung–Kutta method. The parameters

$Ly=5$, $F=1.0$, $\bar{u}=0.7$, $n=10$, $\Delta k=0.75k_0$, $a_0=0.15$, $\gamma=0.4$, $\rho=1.0$ and $\varepsilon=0.24$ are chosen in the present paper. Because dipole block usually occurs at the end of the storm track (the maxima of the high–frequency eddy energy) (Holopainen and Fortelius, 1987), we may take $b=2.87$. For this case, the localized synoptic–scale waves are located $\frac{\pi}{2}$ upstream of the weak, incipient vortex pair block represented by a dipole envelope Rossby soliton at $x=0.0$. According to (29), we can define $Cgm=Cg+dZ/dt$ and $Cpm\approx[\omega-(dP/dt)]/k$ as the group velocity and phase speed of the forced planetary–scale envelope soliton, respectively. Furthermore, $Cgp=Cgm-Cpm$ is defined as a measure of non–dispersion. If Cgp is zero, then the system becomes non–dispersive. But, if Cgp is nonzero, this system is dispersive. Based on this parameter, we can know the role of synoptic–scale waves in forcing dipole block.

Figure 2 shows the numerical solutions of $M(t)$, $K(t)$, $Z(t)$, $Cgm(t)$, $Cpm(t)$ and

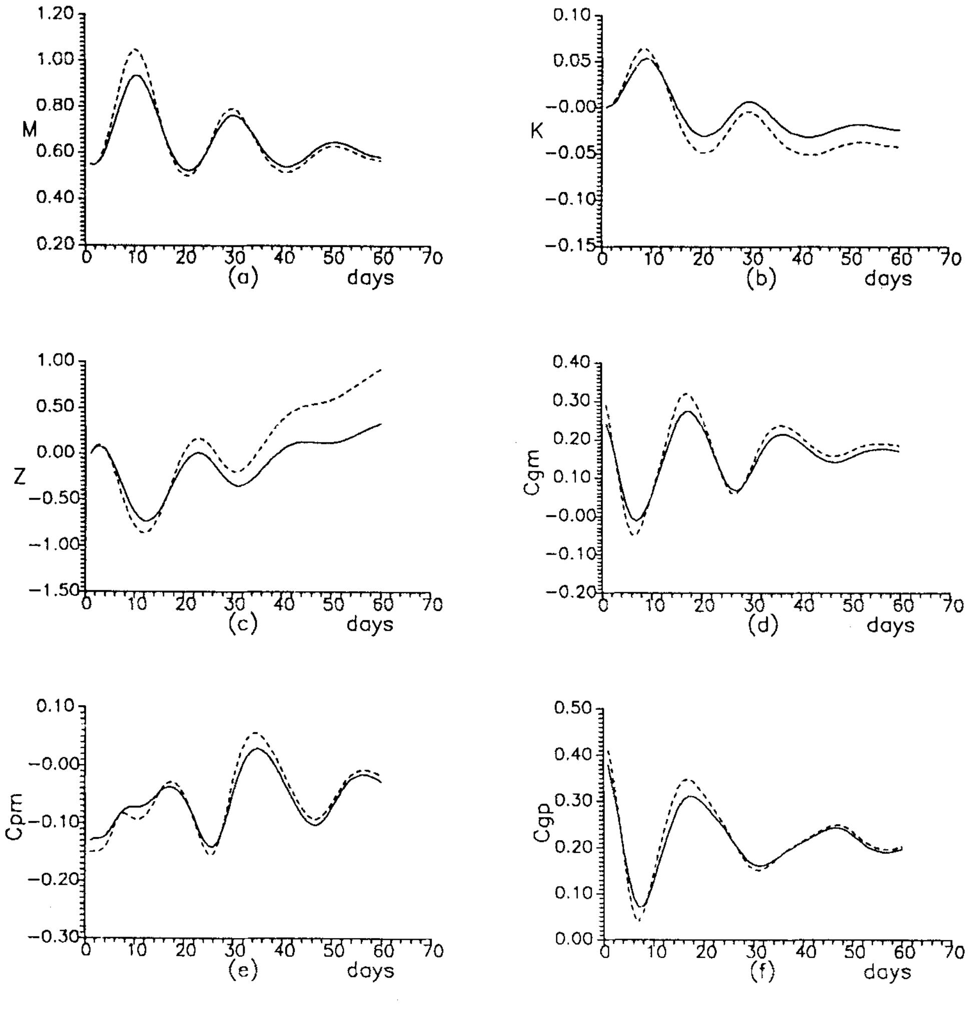

Fig. 2. The numerical solutions of the parameter equations (21)–(24) for the soliton parameters $M(t)$, $K(t)$, $Z(t)$, Cgm, Cp and Cmp for the initial data $M(0)=0.55$, $K(0)=0$, $Z(0)=0$, and $P(0)=0$.

$Cgp(t)$ for parameter equations (25)–(28) when the initial data $M(0)= 0.55$, $K(0)= 0$, $Z(0) = 0$ and $P(0)= 0$ are chosen.

It can be found from Fig. 2 that when the localized synoptic–scale waves propagate downstream and interact with the planetary–scale envelope Rossby soliton, this planetary–scale dipole soliton can be deformed. In this process, for $a_0 = 0.15$ the soliton amplitude $M(t)$ increases from 0.55 at day 0 to 0.93 at day 9, and then it decreases from 0.93 to 0.52 at day 20. This means that under the near–resonant forcing of synoptic–scale waves upstream, the planetary–scale dipole envelope soliton can be transferred from a "high–index" state into a "low–index" state. At the same time, we note that the soliton wavenumber $K(t)$ remains positive before 15 days so that the zonal wavelength of the planetary–scale dipole envelope soliton becomes longer during this period. After day 15, the zonal wavelength of the planetary–scale soliton becomes shorter. On the other hand, we find that both the soliton group velocity and the absolute value of the soliton phase speed decrease so that Cgp becomes small even zero. In this case, the soliton system becomes weak dispersion even non–dispersion ($Cgp = 0$) as the planetary–scale dipole envelope soliton is amplified due to the near–resonant forcing of upstream synoptic–scale waves. This can, to some extent, explain why the synoptic–scale waves upstream can reinforce and maintain dipole block. Consequently, we can infer that when the planetary–scale dipole soliton (incipient block) interacts with the synoptic–scale waves upstream, it can be amplified into dipole block and maintained by these waves. A similar conclusion can be obtained if one chooses other parameters (figures omitted). In contrast, when $\rho < 0$, the synoptic–scale eddies given in (7) will lead to the decay of incipient dipole block. Only when the synoptic–scale waves themselves have a moderate parameter match, they can reinforce and maintain planetary–scale dipole anomaly flow. Colucci (1985) suggested that blocking might be understood as a response of the planetary–scale waves to synoptic–scale perturbations, which act as sources of energy and vorticity for the incipient block. Such a process can be explained in terms of the results obtained from Fig.2. On the other hand, because the second–order synoptic– scale flow ψ'_2 contains the planetary–scale dipole soliton component ψ_0, the localized synoptic–scale waves ψ' can be deformed when the planetary–scale dipole soliton is deformed by the localized synoptic–scale waves. In order to see the role of localized synoptic–scale waves in the onset, maintenance and decay of vortex pair block, the streamfunction fields of both deformed planetary–scale dipole soliton and synoptic–scale waves will be plotted.

Under the same parameter conditions as in Fig.2 the streamfunction fields of the planetary–scale dipole soliton ψ and the synoptic–scale eddies ψ' and the total streamfunction field ($\psi_T = \psi + \psi'$) are described in turn in Figs. 3, 4 and 5.

Fig.3 shows the streamfunction field, ψ, of the planetary–scale dipole envelope Rossby soliton without including synoptic–scale waves, while Fig.4 presents the instantaneous contour plots of the localized synoptic–scale eddies ($\psi' \approx \varepsilon^{3/2}\psi'_1 + \varepsilon^{5/2}\psi'_2$). It is obvious to see that at day 0 this planetary–scale envelope soliton possesses a weak dipole, which can be considered as an incipient block or a zonal flow (Colucci, 1985; Vautard et al., 1988a), but the incipient synoptic–scale waves are located upstream of the weak, incipient dipole block at $x = 0$. The flow pattern of the synoptic–scale waves is in good agreement with the synoptic–scale eddies during the zonal flow observed by Vautard and Legras (1988b, their Fig.9a). When these synoptic–scale waves propagate eastward and encounter the incipient, weak dipole soliton existed downstream, a strong near–resonant interaction take places. This interaction

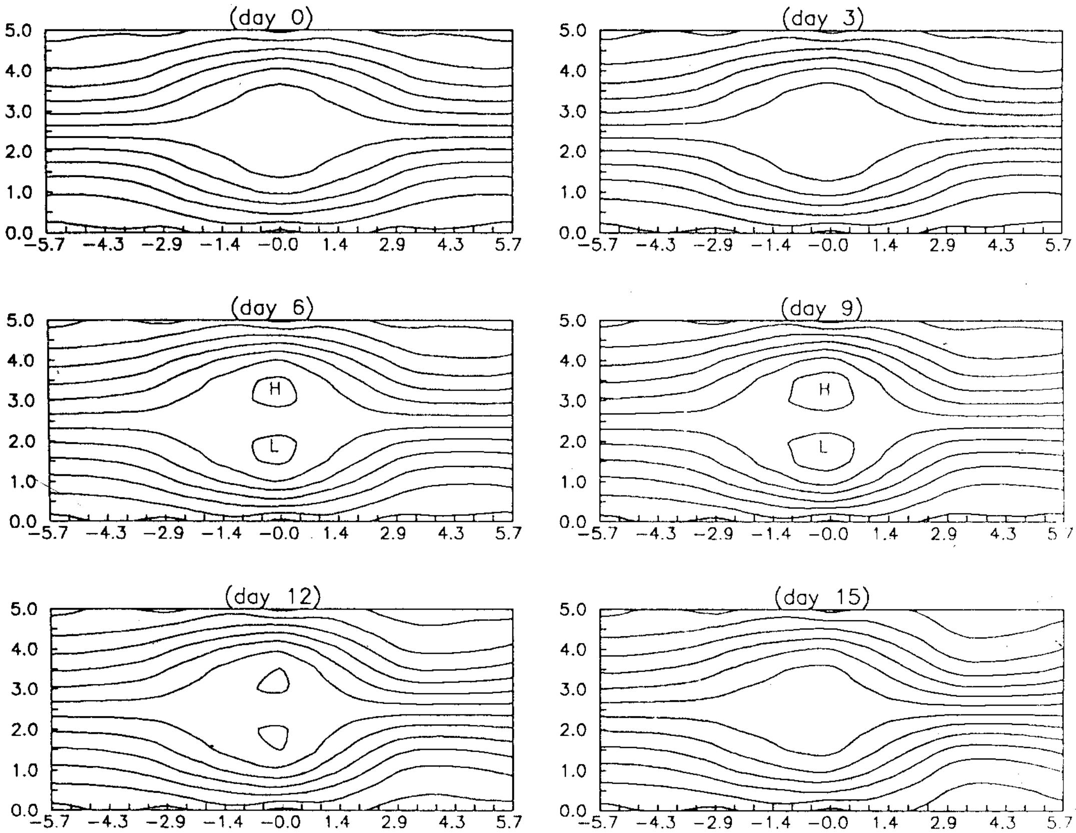

Fig. 3. Time sequence of the instantaneous field ψ of planetary–scale dipole envelope Rossby soliton interacting with the localized synoptic–scale eddies at $x=-2.87$ for the same parameters and initial data as in Fig.2. Contour interval 0.3.

will lead to the deformation of both planetary–scale dipole soliton and synoptic–scale waves. Although previous theories can indicate the role of synoptic–scale waves in forcing dipole blocking (Haines and Marshall, 1987; Malanotte et al., 1987 and Malguzzi, 1993), but cannot provide a clear physical picture of how both planetary–scale wave and synoptic–scale waves interact during the life cycle of blocking. In other words, the problem how the planetary–scale blocking wave is deformed during the passage of synoptic–scale waves upstream is not solved yet. Using our theoretical model presented here, this problem can be solved completely. It is easily found from Figs.3 and 4 that through the near–resonant forcing of synoptic–scale waves, the planetary–scale envelope soliton can be amplified, while the synoptic–scale waves themselves will undergo a strong deformation due to the feedback of amplified soliton block. For example, at day 3 the planetary–scale soliton is slightly intensified, but at days 6 and 9 the intensification of the soliton block is more remarkable. Afterwards, this soliton block begins to weaken. In this process, a noteworthy character is that during the establishment of dipole block the synoptic–scale waves upstream are enhanced and split into two branches: one spreading northeastward along the western flank of the developing anticyclone, and the other southeastward around the cyclonic low–frequency circulation anomaly developing to

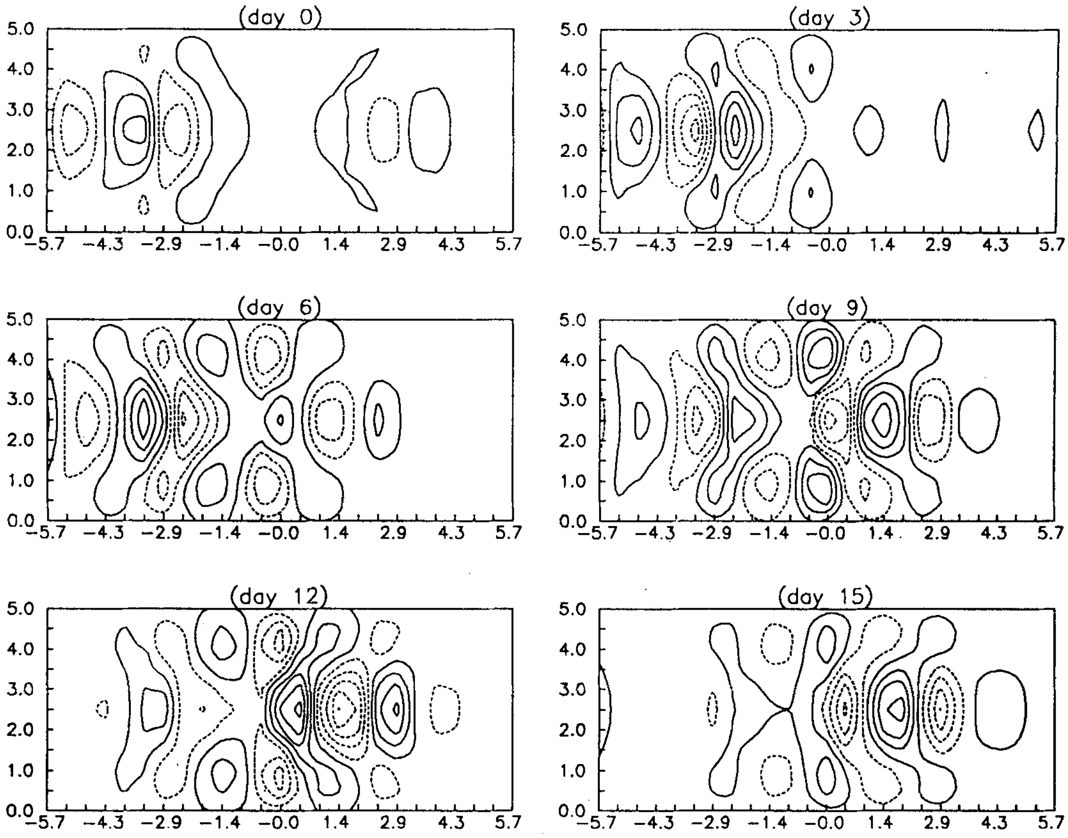

Fig. 4. The instantaneous chart of the streamfunction contours for the localized synoptic–scale eddies including the feedback of the planetary–scale dipole envelope Rossby soliton at 55°N for the same parameters and initial data as in Fig.2. Contour interval 0.3.

the south of the block. This enhancement is very remarkable before 9 days. The synoptic behavior of synoptic–scale waves during the blocking onset being very similar to the result found by Nakamura and Wallace (1990), who observed that the enhancement of synoptic–scale waves tends to occur about five days before the blocking pattern is fully established. Thus, the enhancement and splitting of synoptic–scale waves completely originate from the feedback of amplified dipole block. To some extent, this result may explain why the Atlantic storm track (the maxima of the synoptic–scale eddy energy) is split into two branches when high–frequency (synoptic–scale) eddies are steered around the block (Holopainen and Fortelius, 1987; Vautard et al., 1988a,b). Although diagnostic studies had indicated that synoptic–scale waves could undergo deformation as they approached a blocked area (Shutts, 1983; Holopainen and Fortelius, 1987), how the planetary–scale blocking wave was deformed during the passage of synoptic–scale waves being not understood. However, we can find from Fig.2 that during the passage of upstream synoptic–scale waves, the planetary–scale soliton block downstream can be amplified, and transferred from a dispersion system to a weak dispersion even non–dispersion system. This result is new, which describes how the soliton block is deformed during its interaction with synoptic–scale waves.

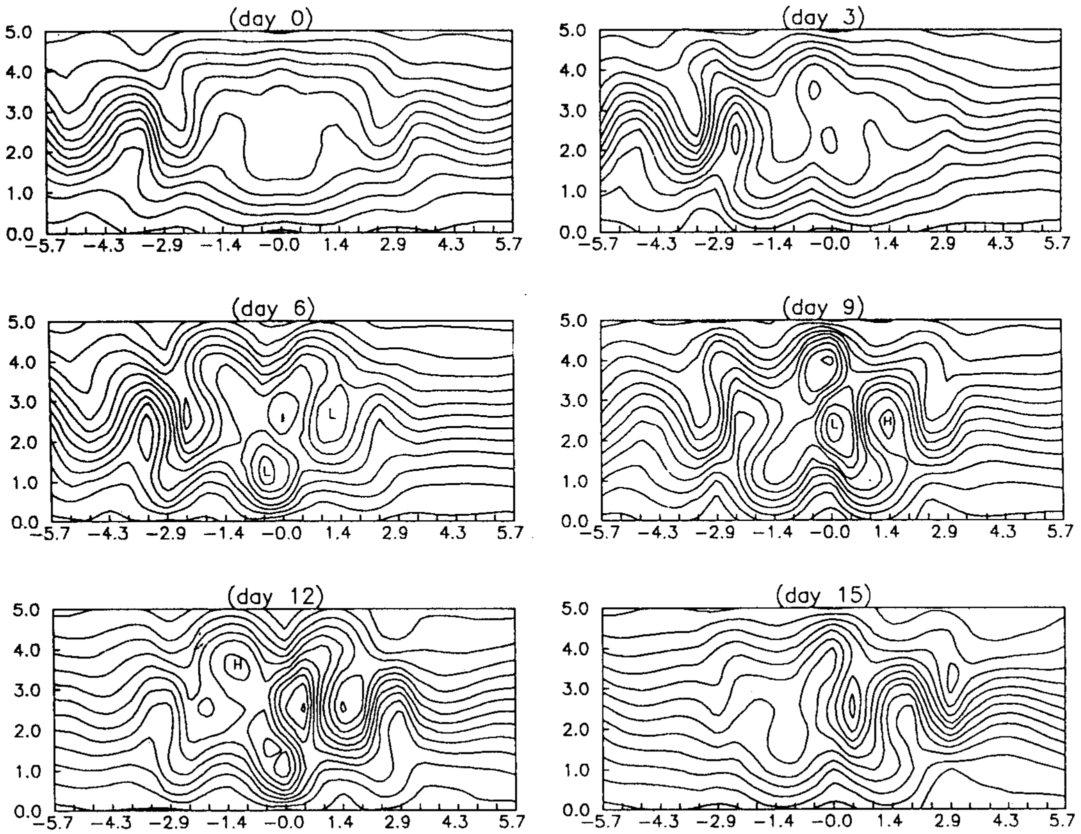

Fig. 5. Time sequence of the instantaneous total streamfunction field ψ_T of the planetary–scale dipole envelope Rossby soliton interacting with the localized synoptic–scale eddies at $x=-2.87$ for the same parameters and initial data as in Fig.2. Contour interval 0.3.

Therefore, the results obtained here can explain why cyclogenesis is particularly intense and planetary–scale waves undergo drastic changes during the onset of blockings during the life cycle of blocking (Nakamura and Wallace, 1990).

Fig. 5 shows sequences of contour plots of the total streamfunction field ψ_T. It is found from this figure that at day 0, a weak block is located at $x=0$, which is regarded as an incipient block (or called zonal flow) (Colucci 1985;Vautard and Legras 1988b), and both small–scale low pressure troughs and high pressure ridges prevail upstream of the weak incipient block. When the small–scale systems (synoptic–scale waves) propagate eastward and interact with the weak incipient block downstream, the small–scale low pressure trough and high pressure ridge are amplified. In this process, the east–west scale of the small–scale vortices seems to undergo a compression. At the same time, the planetary–scale anticyclonic vorticity induced by synoptic–scale waves goes northward, while the planetary cyclonic vorticity goes southward. This process can reinforce the incipient soliton block downstream. This behaviour can be also seen from Fig.6, which describes the fields of $F_p = -J(\psi'_1, \nabla^2\psi'_1)_P$. At day 3 a closed vortex pair block is observed at $x=0$. At the same

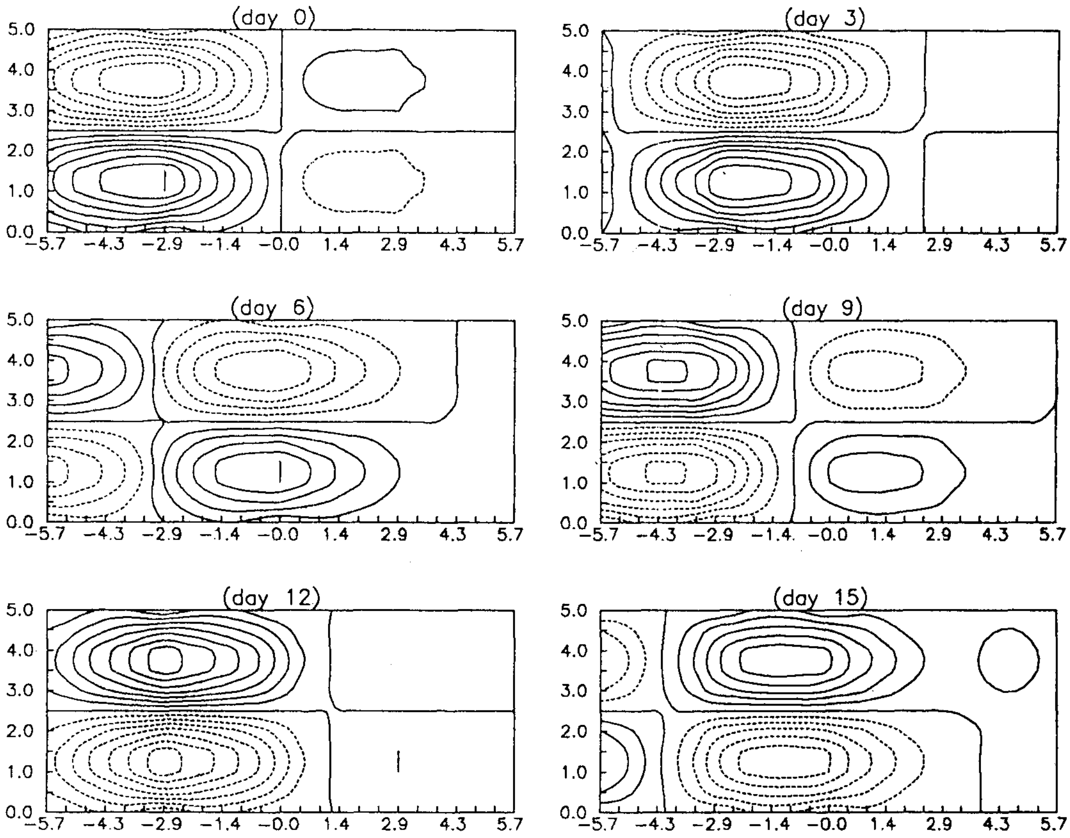

Fig. 6. Time sequence of the instantaneous F_e field induced by the localized synoptic–scale eddies initially at $x=-2.87$ for the same parameters as in Fig.2. Contour interval 0.025.

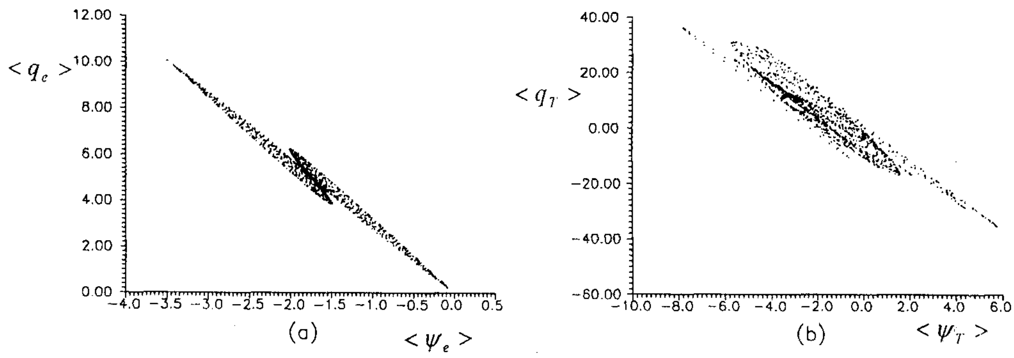

Fig. 7. The variation of the time–averaged potential vorticity with time–averaged streamfunction: (a) $d\langle q_e \rangle / d\langle \psi_e \rangle$; (b) $d\langle q_T \rangle / d\langle \psi_T \rangle$ during the mature period of blocking (from day 6 to day 12).

time, small–scale trough and ridge upstream of the vortex pair block are further intensified. At day 6, a very strong vortex pair block occurs because the negative and the positive planetary–scale vorticities induced by the synoptic–scale waves further enter the anticyclonic and cyclonic regions of the vortex pair block. At day 9, this dipole block becomes more apparent. Afterwards, the dipole block will decay because the field F_p travels eastward and shifts from the dipole block and the synoptic–scale waves also become weak (Fig. 4 at day 12). This result reflects in part that the synoptic–scale waves upstream will become weak as the dipole block becomes weak. It may be associated with such a fact that the feedback of dipole block flow on the synoptic–scale waves becomes weak during the decay of vortex pair block. In contrast, during the onset of dipole block the enhancement of synoptic–scale waves is due to the feedback of enhanced dipole block. Thus, both vortex pair block and synoptic–scale waves possess a symbiotic relation (Cai and Mak, 1990; Nakamura and Wallace, 1990). Previous theoretical models cannot describe such a relation. It is clear to find from Fig.5 that there appear to be several isolated cyclonic or anticyclonic vortices coexisting within the blocking region. This figure is very similar to an observational case found first by Berggren et al. (1949). Very unfortunately, previous theoretical model cannot explain such a structure. During the period of blocking (from day 6 to day 12), $d<q_e>/d<\psi_e>$ and $d<q_T>/d<\psi_T>$ are plotted in Fig. 7, where $<q_e> = \nabla^2 <\psi_e> + \beta y$, $<q_T> = \nabla^2 <\psi_T> + \beta y$, $<\psi_e>$, and $<\psi_T>$ are time averaging during the period from day 6 to day 12.

Strictly speaking, $<q_e>$ and $<\psi_e>$ does not have an exact linear relationship during the period of blocking in which the synoptic–scale waves are filtered out. But, approximately, it can be regarded that $<q_e>$ and $<\psi_e>$ have a linear relationship. However, when the synoptic–scale eddies are included in the blocking region, the linear relationship between $<q_T>$ and $<\psi_T>$ becomes more impossible. This shows that the synoptic–scale waves can cause the breakdown of the linear relationship between time–averaged potential vorticity and streamfunction (Butchart et al., 1989; Mu et al., 1999).

Although synoptic–scale waves here are designed to be very simple, our nonlinear analytical model proposed here can produce the main features of the interaction between the synoptic–scale waves and planetary–scale waves during the life cycle of blocking. It should be pointed out that if the amplitude of the initial planetary–scale envelope soliton (incipient block) is chosen to be too small or the synoptic–scale waves are located downstream of the incipient soliton block, this incipient soliton block cannot be amplified into a blocking system (figures omitted). This point has been confirmed. As suggested by Colucci (1987), the planetary–scale waves might require at least a critical amplitude before the blocking is established. During the onset of blocking there appears to be a positive feedback between the synoptic–scale waves and planetary–scale waves because of higher incipient planetary–scale wave amplitude even though the planetary–scale response to this feedback depends on the location of the synoptic–scale waves relative to the planetary–scale waves. However, such a feedback doesn't occur during the nonblocking period, perhaps it originates from low incipient planetary wave amplitude. On the other hand, we can find that the parameter match of synoptic–scale waves themselves seems to strongly dominate the synoptic–planetary scale wave interaction. For example, when $\rho < 0$, the synoptic–scale waves do not favor the amplification of dipole block even if the incipient planetary–scale wave has a large amplitude (figures omitted). In other words, only under a moderate condition the interaction between planetary– and synoptic– scale waves can reinforce and maintain dipole block.

5. Conclusion and discussions

In this paper, we have proposed a forced envelope soliton model to describe the interaction between the planetary– and synoptic–scale waves in weak background westerly wind during the life cycle of blocking. It can be shown that for a moderate parameter match of synoptic–scale eddies themselves when these eddies impinge upon the weak dipole block from the west, the weak block can be amplified into a very strong vortex pair block and the eddies are split into two branches around the block. In addition, we can see that the forcing of localized synoptic–scale eddies is able to make the vortex pair flow (dipole envelope Rossby soliton here) weak dispersive even nondispersive. This may explain the establishment and maintenance of vortex pair blocks by localized synoptic–scale eddies. On the other hand, it is also clear that when the vortex pair block weakens considerably, the deformation of the synoptic–scale eddies does also weaken, indicating that both of them have a symbiotic relation. The main conclusions are:

(1) The forcing of the synoptic–scale eddies is found to increase the amplitude of the vortex pair structure described by an envelope soliton and decrease its group velocity and phase speed, that is, the dispersive vortex pair system may become weak dispersive even nondispersive by adding the forcing of synoptic–scale eddies. This can explain why the synoptic–scale eddies can reinforce and maintain dipole blocking. The onset and maintenance of vortex pair block by synoptic–scale eddies are of transfer process from dispersive system to weak dispersive even nondispersive system. During the mature period of blocking, the time–averaged blocking structure can be approximately considered as a nondispersive modon solution (McWilliams, 1980), but during the onset and decay period of blocking it cannot be regarded as a nondispersive modon solution. Consequently, the forced envelope Rossby soliton model proposed here seems to be more reasonable for blocking events than the KdV soliton and modon models.

(2) The result about the interaction between vortex pair block and synoptic–scale eddies obtained in the forced envelope soliton model is found to be very similar to the life cycle of blocking observed by Berggren et al. (1949). This shows from another aspect that the envelope Rossby soliton is reasonable for blocking events.

On the other hand, it should be pointed out that although the leading–order part (the first–order synoptic–scale flow) of the synoptic–scale eddies is designed to consist of a pair of sideband perturbations with the time scale less than one week and is assumed to be placed in the upstream of the dipole flow, the second–order synoptic–scale flow can be induced by the interaction between the first–order synoptic–scale flow and the dipole block flow. In this case, the synoptic–scale eddies (the sum of the first–and second–order synoptic–scale flows) are found to be deformed when interacting with the dipole flow. In spite of the simplicity, the result obtained in our model compares rather well with observations in many respects. Even so, the model of higher vertical resolution and a more realistic mean wind profile should be considered if better agreement is expected to obtain. Comparing with the previous theoretical works (Pierrehumbert and Malguzzi, 1984; Haines and Marshall, 1987), our model seems to be another candidate to investigate the interaction between the traveling weather systems and planetary waves during the life cycle of blocking (see the observational evidence given by Berggren et al. 1949, their Figs. 14–18 or Fig.26). Another characteristic of this model is the absence of large–scale topography. This omission is acceptable in studying the dynamics of blocking over the Atlantic. Although our model is simple, our approach provides a basis for investigating the interaction between synoptic–scale eddies and blocking flow in the mid–high latitudes. The role of baroclinic synoptic–scale eddies in forcing blocking circulation has been investigated in Luo (2000).

In addition, the comparison with observed data should be made. The study will be presented in another paper.

Appendix

The coefficients Q_i, p_i, r_i, s_i and $h_i (i=1,2)$ in (14) are defined by

$$p_i = \frac{k-2k_i}{(\beta+F\bar{u})[k_i+k]-[\bar{u}(k_i+k)-(\omega_i+\omega)][(k_i+k)^2+\frac{9m^2}{4}+F]},$$

$$r_i = \frac{k+2k_i}{(\beta+F\bar{u})[k_i+k]-[\bar{u}(k_i+k)-(\omega_i+\omega)][(k_i+k)^2+\frac{m^2}{4}+F]},$$

$$s_i = \frac{k+2k_i}{(\beta+F\bar{u})[k_i-k]-[\bar{u}(k_i-k)-(\omega_i-\omega)][(k_i-k)^2+\frac{9m^2}{4}+F]},$$

$$h_i = \frac{k-2k_i}{(\beta+F\bar{u})[k_i-k]-[\bar{u}(k_i-k)-(\omega_i-\omega)][(k_i-k)^2+\frac{m^2}{4}+F]},$$

and $Q_i = k^2+m^2-(k_i^2+\frac{m^2}{4}) \quad (i=1,2)$.

REFERENCES

Benney, D. J., 1979: Large–amplitude Rossby waves. *Stud. App. Math.*, **60**, 1–10.

Berggren, R., B. Bolin, and C. G. Rossby, 1949: An Aerological Study of zonal motion, its perturbations and break–down. *Tellus*, **2**, 14–37.

Butchart, N., K. Haines, and J. C. Marshall, 1989: A theoretical and diagnostic study of solitary waves and atmospheric blocking. *J. Atmos. Sci.*, 2063–2078.

Cai, M., and M. Mak, 1990: Symbiotic relation between planetary and synoptic–scale waves. *J. Atmos. Sci.*, **47**, 2953–2968.

Colucci, S. J., Loesch A. Z. L. F. Bosart, 1981: Spectral evolution of a blocking episode and comparison with wave interaction theory. *J. Atmos. Sci.*, **38**, 2092–2111.

Colucci, S. J., 1985: Explosive cyclogenesis and large–scale circulation changes: Implications for atmospheric blocking. *J. Atmos. Sci.*, **42**, 2701–2717.

Colucci, S. J., 1987: Comparative diagnosis of blocking versus nonblocking planetary–scale circulation changes during synoptic–scale cyclogenesis. *J. Atmos. Sci.*, **44**, 124–139.

Crum, F. X., and D. E. Stevens, 1988: A case study atmospheric blocking using isentropic analysis. *Mon. Wea. Rev.*, **116**, 223–241.

Dole, R. M., 1986: Persistent anomalies of the extratropical Northern Hemisphere winter time circulation: structure. *Mon. Wea. Rev.*, **114**, 178–207.

Frisius, T., Lunkeit, F., Fraedrich, K., James, I. N., 1998: Storm–trackorganization and variability in a simplified atmospheric global circulation model. *Q. J. R. Meteor. Soc.*, **124**, 1021–1043.

Green, J. S. A., 1977: The weather during July 1977: Some dynamical consideration of the drought. *Weather*, **32**, 120–126.

Hansen, A., and T. C. Chen, 1982: A spectral energetics analysis of atmospheric blocking. *Mon. Wea. Rev.*, **110**, 1146–1159.

Haines, K., and J. C. Marshall, 1987: Eddy–forced coherent structures as a prototype of atmospheric blocking. *Quart. J. Roy. Meteor. Soc.*, **113**, 681–704.

Hasegawa, A., and Y. Kodama, 1995: *Solitons in optical communications*, Clarendon Press, 320pp.

Holopainen, E., and C. Fortelius, 1987: High–frequency transient eddies and blocking. *J. Atmos. Sci.*, **44**, 1632–1645.

Illari, L., and J. C. Marshall, 1983: On the interpretation of eddy fluxes during a blocking episode. *J. Atmos. Sci.*, **40**, 2232–2242.

Ilarri, L., 1984: A diagnostic study of the potential vorticity in a warm blocking anticyclone. *J. Atmos. Sci.*, **41**, 3518–3526.

Jeffrey, A., Kawahara, T., 1982: *Asymptotic Methods in Nonlinear Wave Theory*. Pitman Press, 273pp.

Lejen S, H., and H, Kland, 1983: Characteristics of Northern Hemisphere blocking as determined from a long series of observational data. *Tellus*, **35A**, 350–362.

Lupo, A., R., and P.J. Smith, 1995: Planetary and synoptic–scale interactions during the life cycle of a mid–latitude anticyclone over the North Atlantic. *Tellus*, **47A**, 575–596.

Luo Dehai, 1999a: Near–resonant topographically forced envelope Rossby solitons in a barotropic flow. *Geophys. Astrophys. Fluid Dyn.*, **90**, 161–188.

Luo Dehai, 1999b: *Envelope Rossby solitons in the large–scale atmosphere and blocking circulations*, China Meteorological Press, 113pp. (in Chinese)

Luo Dehai, 2000: Planetary–scale baroclinic envelope Rossby solitons in a two–layer model and their interaction with synoptic–scale eddies. *Dyn. Atmos. Oceans*, **32**, 27–74.

Malanotte–Rizzoli, P., and P. Malguzzi, 1987: Coherent structures in a baroclinic atmosphere. Part III: Block formation and eddy forcing. *J. Atmos. Sci.*, **44**, 2493–2505.

Malguzzi, P., 1993: An analytical study on the feedback between large– and small–scale eddies. *J. Atmos. Sci.*, **50**, 1429–1436.

McWilliams, J. C., 1980: An application of equivalent modons to atmospheric blocking. *Dyn. Atmos. Oceans*, **5**, 219–238.

Mu Mu, Wu Yonghui, Tang Mozhi, and Liu Haiyan, 1999: Nonlinear stability analysis of the zonal flows at middle and high latitudes. *Adv. Atmos. Sci.*, **16**, 569–580.

Mullen, S. L., 1987: Transient eddy forcing of blocking flows. *J. Atmos. Sci.*, **44**, 3–22.

Nakamura, H., and J. M. Wallace, 1990: Observed changes in baroclinic wave activity during the life cycles of low–frequency circulation anomalies. *J. Atmos. Sci.*, **47**, 1100–1116.

Nakamura, H., and J. M. Wallace, 1993: Synoptic behaviour of baroclinic eddies during the blocking onset. *Mon. Wea. Rev.*, **121**, 1892–1903.

Nakamura, H., Nakamura, M., and J. L. Anderson, 1997: The role of high– and low–frequency dynamics in blocking formation. *Mon. Wea. Rev.*, **125**, 2074–2093.

Neilley, P. P., 1990, Interaction between synoptic–scale eddies and the large–scale flow during the life cycles of persistent flow anomalies, Ph. D. dissertation, MIT, 272pp.

Pierrehumbert, R. T., and P. Malguzzi, 1984: Forced coherent structures and local equilibria in a barotropic atmosphere. *J. Atmos. Sci.*, **41**, 246–257.

Rex, D. F., 1950: Blocking action in the middle troposphere and its effet uponregional climate. I: An aerological study of blocking action. *Tellus*, **2**, 196–301.

Robinson, W. A., 1991: The dynamics of low–frequency variability in a simple model of the global atmosphere. *J. Atmos. Sci.*, **48**, 429–441.

Shutts, G. J., 1983: The propagation of eddies in diffluent jetstreams: eddy vorticity forcing of blocking flow fields. *Quart. J. R. Meteor. Soc.*, **109**, 737–761.

Taniuti,T., and K. Nishihara, 1983: *Nonlinear waves*, Pitman Publishing Limited, 258.

Tsou, C. S., and P. J., Smith, 1990: The role of synoptic / planetary–scale interaction during the development of a blocking anticyclone. *Tellus*, **42A**, 174–193.

Tung, K. K., and A. S. Lindzen, 1979: A theory of stationary long waves. Part I: A simple theory of blocking. *Mon. Wea. Rev.*, **107**, 714–734.

Vautard, R., B. Legras and M. Deque, 1988a: On the source of midlatitude low–frequency variability. Part I: A statistical approach to persistence. *J. Atmos. Sci.*, **45**, 2811–2843.

Vautard, R., and B. Legras, 1988b: On the source of midlatitude low–frequency variability. Part II: Nonlinear equilibration of weather regimes. *J. Atmos. Sci.*, **45**, 2845–2867.

Yamagata, T., 1980: The stability, modulation and long wave resonance of a planetary wave in a rotating, two–layer fluid on a channel beta–plane. *J. Meteor. Soc. Japan*, **58**, 160–171.